Müller, and ending with those now generally in use, is excellent; although it is evident from what is here laid down, that our classification of micro-organisms is as yet to a large extent empirical, and that there is great need for a classification constructed on a thoroughly sound and scientific basis. The principles of sterilisation and pure cultivation are given succinctly but very clearly. The section devoted to the heat-resisting bacteria, their place in nature, and their importance in the fermentation and food-stuff industries, is one of considerable interest. The principal organisms in this group are described as the *Bacillus subtilis* and its congeners, the *Clostridium butyricum*, the genus Granulobacter, and various other organisms associated with the butyric acid fermentation, the fermentation of cellulose, the "retting" of flax and hemp, and the production of rancidity of fats.

The relation of the study of the life-history of these various organisms to the preservation of milk, meat, eggs, vegetables and fruit is fairly carefully considered, as are also the lactic fermentation and the allied decompositions, special stress being laid on the production of optically active organic compounds by fermentation, on the artificial souring of cream, the coagulation of milk, and on the importance of the part that various lactic acid bacteria play in the processes of distilling, brewing and vinification; and in the preparation of fodder, the making of brown hay into sweet ensilage and sour fodder. Then the work done by bacteria in tanning, in the manufacture of sugar in the conditions known as "ropiness" in milk, wine, beer, and other liquids are all somewhat fully and interestingly treated. A special section is devoted to the decomposition and transformation of organic nitrogenous compounds; this, of course, constitutes a very important part of the work, and, in conjunction with the section on oxidising fermentation, affords a very large amount of information on the bacterial processes involved in the breaking up of various organic compounds. It is interesting to note how closely these processes are associated with those of fermentation of cheese and of similar proteid substances.

Altogether this volume, the first of two, is an exceedingly interesting and valuable contribution to the study of technical mycology. The work of translation is well done, but there are one or two slips which might with advantage be corrected in future editions: for instance, "typhus" is throughout used for "typhoid," this, of course, being a literal translation of the German *typhus* without the term *abdominalis*, which is always added to indicate our *typhoid* fever. It need scarcely be mentioned that the work will probably be hailed by English workers with gratitude, but we may point out that the term "mycology" will convey to the general reader very little idea as to the scope of the work. Many years ago a work was published in this country to which the title "Pathological Mycology" was given, a work which was largely overlooked because of its title. Since then this same title has been used abroad, where the significance of the word appears to be more fully appreciated. We think the translator would have been wise had he selected some title more generally "understanded of the people" for what, after all, must to a certain extent be a popular work. There will, however, be a considerable demand for this book amongst those who are engaged in pathological and technical bacteriology, who, of course, will appreciate both the title and the work; but the translator must expect to find that some, at least, of his possible readers will pass over this book simply because they do not understand the title.

Messrs. Griffin have done their part in a thoroughly workmanlike fashion, and we congratulate both author and translator in having their work placed so well before the reading public.

PARTIAL DIFFERENTIAL EQUATIONS.

Leçons sur l'intégration des équations aux derivées partielles du second ordre à deux variables indépendantes. Par E. Goursat. T. I. pp. viii+226; T. II. pp. 344. (Paris: A. Hermann, 1897, 1898.)

A DIFFERENTIAL equation, in its usual form, states an analytical problem with a certain assumption as to the form of the answer. It implies the existence of a dependent variable, capable of being differentiated so far as the order of the equation indicates; and the solution of the equation consists in discovering a relation among the variables, free from differential coefficients, such that the given differential equation may be derived from it. The question at once arises: what is the most comprehensive form of solution? Is it possible in every case to define an integral relation connecting the variables equivalent to the differential equation in the sense that not only is the differential equation derivable from it, but every possible relation consistent therewith is included as a particular case in the integral equation? In the early days of the infinitesimal calculus it was observed that ordinary differential equations could be obtained by eliminating constants; while partial differential equations could be derived by the elimination of constants or of arbitrary functions. In some cases the reverse process of starting with the differential equation and arriving at an integral relation, involving arbitrary constants or functions, or both, was found to be practicable; and it came to be taken for granted that integral relations of this kind always existed, the only difficulty being that of discovering them.

But, with the advance of function-theory, the peculiar difficulties of the subject have gradually become more evident. It is true that, with regard to ordinary differential equations and partial differential equations of the first order, the general form of solution has been established, and the hypothesis of the earlier mathematicians justified; but when we come to partial differential equations of the second and higher orders, the aspect of the problem is radically changed. In most cases it is hopeless to attempt to assign an explicit form of the general integral, or even to prove its existence; and we have to content ourselves with the study of solutions subject to certain special limitations. Thus we have the problem of Dirichlet in the theory of potential; or again the problem of Cauchy, which forms the leading idea of M. Goursat's original and fascinating treatise.

To explain what this means, let us take the case of an equation of the second order with two independent variables, say $\phi(x, y, z, p, q, r, s, t) = 0$, the notation being as usual. Assume x, y, z, p, q functions of a single variable, subject solely to the condition $dz = pdx + qdy$; we thus have a multiplicity of one dimension, which may be called an orientation of the first order. In general by means of $\phi = 0$, the relations $dp = rdx + sdy$, $dq = sdx + tdy$, and those derivable from them by differentiation, it is possible to find a definite expansion for z, which formally satisfies the differential equation and also contains the given orientation: if the expansion is convergent in a certain domain, this defines z as an analytical function of x, y. Geometrically, if we take x, y, z as point coordinates, the assumed orientation consists of an arbitrary curve, with an arbitrary, but continuous, distribution of tangent planes along it, enveloping a developable surface; if we like, we may regard it as a thin ribbon cut out of a developable. The process sketched above is equivalent to finding an integral surface containing the aforesaid ribbon, or in other words containing the given curve, and touching at each point of it the given associated tangent plane. The problem of Cauchy for an equation of the second order is to find a solution capable of being specialised, by the choice of arbitrary constants or arbitrary functions, or both, so as to contain any given orientation of the first order. Such a solution is said to be general in Cauchy's sense, as distinguished, for example, from one that is general according to Ampère's celebrated definition.

It may happen that the orientation of the first order, M_1 say, is such that the relations $\phi = 0$, $dp = rdx + sdy$, $dq = sdx + tdy$ are, for every element of it, equivalent to only two independent equations; in this case Cauchy's problem becomes indeterminate, and there are an infinite number of integral surfaces containing M_1, which is then said to be a characteristic of the first order of $\phi = 0$. It is an exception for an equation of the second order to admit of a multiplicity M_1; since

$$\phi\left(x, y, z, p, q, \frac{dp}{dx} - s\frac{dy}{dx}, s, \frac{dq}{dy} - s\frac{dx}{dy}\right) = 0$$

has to be satisfied identically for all values of s, and this leads to a number of distinct relations, not generally compatible. One of these is always

$$\frac{\partial\phi}{\partial r}dy^2 - \frac{\partial\phi}{\partial s}dxdy + \frac{\partial\phi}{\partial t}dx^2 = 0; \ . \ . \ .$$

on every integral surface this equation defines a system of characteristic curves.

Throughout the whole treatise the theory of characteristics plays a predominant part. Thus in Chapters i.–iii., which deal with the equation of Monge and Ampère ($Hr + 2Ks + Lt + M + N(rt - s^2) = 0$), it is shown with admirable clearness how the success of Monge's method of integration depends upon finding integrable combinations of the differential equations of the characteristics. The cases of partial or total failure are discussed as well as those of success; and the reader thus becomes familiar with the *rationale* of the process, instead of merely acquiring facility in applying a method which, in some way that he hardly understands, leads (with good luck) to the required solution. Chapter iii., in particular, contains a large number of important applications very fully worked out.

M. Goursat's first volume concludes with an important chapter on the general theory of characteristics and on intermediate integrals. The notion of characteristics is extended to the second and higher orders, and it is shown, among other things, (1) that every equation of the second order possesses in general two distinct systems of characteristics of the second order; (2) that two characteristics of the second order belonging to two distinct systems, and having in common an element of the second order determine one, and only one, integral surface (p. 193). All equations of the second order may be arranged in four classes according as they have (1) two different systems of characteristics, each of the second order (this is the general case); (2) two systems, one of the first order, one of the second; (3) two systems, usually distinct, each of the first order; (4) one system of the first order.

The second volume begins with an account of Laplace's method of treating linear equations, which may be profitably compared with the discussion of the same subject in Darboux's "Théorie des Surfaces." After this come two chapters, of the highest interest and importance, on systems in involution and on Darboux's method of integration. The first of these deals with systems of equations which admit of solutions involving an infinite number of arbitrary constants, and introduces us to ideas of great value and generality which have been developed by various mathematicians, including M. Goursat himself. The chapter is, to a great extent, introductory to the one on Darboux's method, which immediately follows, and which will probably be found the most engrossing part of the work. The leading idea is that of finding integral combinations of the differential equations of characteristics; not necessarily of the first order, as in Monge's method, but of the second, third, or higher order: thus, for instance, Liouville's equation $s = e^z$ is completely integrated by proceeding as far as the characteristics of the second order. M. Goursat very justly remarks that Darboux's method is the most powerful as yet available, and includes most others, for instance those of Monge, Ampère, and Laplace, as particular cases. In order that it may succeed, it is necessary that every integral of the proposed equation should also be an integral of another partial differential equation which has in common with the given equation an infinity of integrals depending on an arbitrary function, while at the same time the second equation must not be satisfied by all the integrals of the first (II. p. 190). The main practical difficulty is that it is generally impossible to say beforehand whether a given equation admits of solution by this method or not. By means of Lie's theory of transformation-groups it is, however, possible to construct a variety of equations to which Darboux's method may be successfully applied.

The next chapter deals with equations of the kind called by Ampère those of the first class; this is followed by one on transformations; and the treatise concludes with a somewhat miscellaneous chapter containing various generalisations of the preceding theory.

A work so attractive as this, and written by an author so well known, is assured of the favourable reception which it thoroughly deserves; taken with M. Goursat's previous work on equations of the first order, and M. Darboux's "Théorie des Surfaces," it will provide mathematical students with an excellent guide to what has been done in this part of analysis. One way, amongst many others, in which M. Goursat's treatises are likely to be very useful is in giving practical illustrations of Lie's methods. Lie's colossal work on transformation-groups is so very abstract and, at the same time, so exhaustive that it must, we fear, repel the great majority of readers; still it is hardly rash to predict that his ideas, as time goes on and they become more familiar, will prove to be of extreme value and fertility, and profoundly affect, not only the theory of differential equations, but almost every branch of analysis. It should be added that M. Goursat points out that Ampère employed contact transformations of a general character more than seventy years ago; and it is, in fact, one of the author's objects to recall attention to Ampère's remarkable memoirs in cahh. 17, 18 of the *Journal de l'École Polytechnique.* G. B. M.

OUR BOOK SHELF.

Our Weights and Measures: a Practical Treatise on the Standard Weights and Measures in use in the British Empire, with some Account of the Metric System. By H. J. Chaney. Pp. viii + 164. (London: Eyre and Spottiswoode, 1897.)

THE Superintendent of Weights and Measures gives in this book an authoritative account of the present practice in regard to the various weights and measures used in trade or for the purposes of manufacture. The origin and history of ancient systems are briefly traced so far as to show how our present system comes to be what it is, and references are carefully given to other treatises and to Acts of Parliament on all points of importance.

The book is well illustrated. Some of the views are of antiquarian interest: *e.g.* the beautiful pictures showing the interior of the Pyx Chapel at Westminster Abbey, a depository for standards since the Norman period; but most of the illustrations have reference to weights and measures in actual use, and to the arrangements for their inspection and verification. Local inspectors of weights and measures will no doubt look on this book as a very useful and, indeed, indispensable compendium.

Teachers and writers of books on arithmetic would do well to take to heart the remarks on pp. 112–114. Thus not only is a list given of those weights and measures which alone need be taught to the exclusion of various customary and local designations which, from a national point of view, are now obsolete, but it is well pointed out that a few hours' actual weighing and measuring would make the children in schools more at home with standard weights and measures than many hours of bare learning of the tables.

The last section of the work is on weights and measures used for special purposes; it includes, for instance, an account of engineers' gauges and standards, and gives tables of particulars of the Birmingham wire gauge, Whitworth's and Seller's screw threads, the B.A. small screw gauge, and several other standard gauges.

Practicum der Wissenschaftlichen Photographie. By Dr. Carl Kaiserling. Pp. xii + 404. (Berlin: Gustav Schmidt, 1898.)

IN this volume of about 400 pages we have a work which will be read by most photographers, whether amateur or professional, who are familiar with the German language, for, besides covering a great deal of ground, the subject is treated of in much detail. Although portraiture and landscape photography are included in the text, the author presents the subject more especially for those who employ photography as a means of aiding them in their scientific investigations. Thus, for instance, the medical man is enlightened as to the best means of illuminating portions of the human body to get the best effects from his point of view, and to photograph with success anatomical sections for demonstrations or collections. Microphotography is also treated at some length, and is well illustrated by some fine autotypes.

It must not be assumed that the optics and manipulations are here somewhat ignored at the expense of the new lines on which the book has been written. Both of these come in for their full share, and are well discussed and described, besides being copiously illustrated. Most of the new lenses are referred to at some length, and are accompanied by numerous tables for determining the lengths of exposures under different conditions. Methods of obtaining positives and enlargements, stereoscopic photography, Röntgen photography, and photography in natural colours, besides processes for reproduction, are all in their turn dealt with individually; and the reader who wishes to specialise in any one or more of these branches will find ample information in these chapters.

Enough, perhaps, has been written to show that this book is not only a useful *vade-mecum* for the student of science who wishes to obtain the best results in his special line of work, but is a valuable addition to our photographic literature. The illustrations are numerous, and there is, what is often absent from a great many German books, a good index.

Principles of Mechanism: a Treatise on the Modification of Motion by means of the Elementary Combinations of Mechanism, or the Parts of Machines, for use in College Classes, by Mechanical Engineers, &c. By S. W. Robinson, C.E., D.Sc., till recently Professor of Mechanical Engineering in the Ohio State University. Pp. xv + 309. (New York: John Wiley and Sons. London: Chapman and Hall, Ltd., 1896.)

THE main value of this work may not unfairly be said to consist in its 350 illustrations of elementary combinations of mechanism (in many cases more curious than useful), and in the descriptions appended thereto. In regard to the scope of the book, and, it may be added, the degree of clumsiness of expression of which the author is capable, we may quote the second sentence of the introduction: "In Principles of Mechanism we find the application to machines, of the principles of kinematics, or cinematics, the elementary combinations of mechanism of which machines, being studied separately."

A good many rules, useful in the drawing office, are exemplified; but the fundamental principles on which they are based are for the most part left unnoticed. Thus in Fig. 297 we have a complicated drawing of the fixed and moving centrodes of certain mechanisms, but we search the book in vain for any demonstration of the method of instantaneous centres on which the construction depends.

In Fig. 301 the curves of velocity-ratio of crank and connecting-rod are shown: the accompanying description identifies them with the fixed and moving centrodes of the motion. There is no appeal to the fundamental principles involved. In fact the book before us, however suitable for reference by an inventor, seems to us quite unfit for a student's text-book.

Introduzione allo Studio dei Silicati. By Dr. E. Ricci. (Milan: Ulrico Hoepli, 1898.)

IN this pamphlet the author seeks to classify the complex group of the mineral silicates, and he claims for his arrangement the merit of simplicity. The distinction between the hydrous and anhydrous silicates is abandoned, and all mineral silicates are grouped in the two primary divisions of orthosilicates and metasilicates. As simple orthosilicates he includes zircon, phenacite, and willemite with the Peridot family (olivine, sepiolite and calamine); and with the double orthosilicates he